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Optimization of Fuzzy Bi-Index Transshipment Problem

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Abstract
This article presents a method that deals with bi-index transshipment problem. A ranking method based on the point of intersection of diagonals of a generalized trapezoidal fuzzy number is applied to the fuzzy elements of a transshipment problem. An improvisation to a bi-index fuzzy transshipment problem has been done via a differently defined ranking method.

KEYWORDS
Transshipment Problem; Bi-Index; Optimization; Generalized Trapezoidal Fuzzy Numbers; Ranking

1. Introduction

For solving a transportation problem (TP), it is essential to determine the best possible shipping schedule of articles from sources to destinations (sinks), which is the optimum shipping schedule that minimizes the total cost of transportation incurred. The demands of quantities of a commodity at various destinations are met by shipping the units to them from various sources according to the available supplies. An TP requires that an article be shipped to a destination directly from a source, with no stoppages or transfer points in between. On the other hand, transshipment problems make room for the fact that shipping articles through transitory points, rather than shipping them from sources directly to destinations, could in fact result in shipping schedules that result in considerably lower transportation costs. The outcome of making allowance for one or more transitory points between sources and sinks is obtaining a transportation cost that is usually lower than that obtained from the optimum shipping schedule of a regular TP. The transshipment problem is an optimization problem that is more commonly applicable and observed now than it was before. In a transshipment problem; the unit transportation costs, availabilities at sources, requirements at sinks/destinations may be incorporated as fuzzy in nature rather than the traditional approach of considering elements which are deterministic in nature. Newer approaches to solving transshipment problems of this nature are necessary for better optimized

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solutions.

2. Literature Review

Transshipment problems are considered with as well as without the fuzzy approach. In order to optimize transshipment costs in intuitionistic fuzzy environments of Type 1 and Type 2, Kumar et al. (2024) have applied the dynamic ranking function; a pioneering attempt, as mentioned by the authors. A ranking method based on the centroid formula proposed by S.-J. Chen and S.-M. Chen (2007) which has been applied to solve a fuzzy transshipment problem by Gani et al. (2011); dealt with two methods of solving a transshipment problem, namely fuzzy Vogel approximation method (VAM) and the two-phase method; both of which have been used to arrive at the optimal solution. It has been stated by Gani et al. (2011); that S.-J. Chen and S.-M. Chen (2007) overcome the shortcomings of many previously proposed classification methods. P. K. Giri et al. (2015) studied a fixed charge multi-item TSP, under a fully fuzzy environment; and have proposed a few methods to arrive at the fuzzy allocations. They have provided solutions and illustrated the developed models through numerical examples, and have further developed a dominate-based genetic algorithm (P. k. Giri et al. 2018) which has been applied to obtain a solution to fixed-charge multi-item solid transportation problems in a fuzzy environment. Baidya and Bera (2019) have formulated a solid TP in a fully fuzzy environment to maximize profit ; and five new approaches are proposed to defuzzify a model. P. K. Giri et al. (2012) proposed three fuzzy random solid transportation problems , in which fuzziness and randomness are considered simultaneously in times and costs. VIMALA (2016) studied the fuzzy transhipment problem with the 'magnitude technique' used for defuzzification; and all costs considered in this article are symmetric fuzzy numbers. Baskaran and Dharmalingam (2016) have incorporated fuzziness into the unbalanced transshipment problem in the presence of budgetary constraints; where demand and budget are not precisely defined.

3. Methodology

The authors, upon having come across a recently developed method of ranking trapezoidal fuzzy numbers (Patil et al. 2025) have identified the corresponding scope and opportunity of applying this method to optimize a fuzzy bi-index transshipment problem. The objective of the authors in their current work is to establish an improvisation in the solution to a fuzzy transshipment problem by recommending the usage of this new ranking method. In this section; the relevant preliminaries, methods, techniques, and steps involved in solving a fuzzy transshipment problem are discussed.

3.1. Generalized Trapezoidal Fuzzy Number (GTFN):(Patil et al. 2025)

$\tilde{A} = (a_1, a_2, a_3, a_4; w_A)$ with $0 \leq w_A \leq 1$ can be defined as a GTFN $\mu_A(x)$ (Supriya and Gadekallu 2023) given below:

$$\mu_A(x) = \begin{cases} 0 & x < a_1 \text{ or } x > a_4 \\ w_A\left(\dfrac{x-a_1}{a_2-a_1}\right) & a_1 \leq x \leq a_2 \\ w_A & a_2 \leq x \leq a_3 \\ w_A\left(\dfrac{a_4-x}{a_4-a_3}\right) & a_3 \leq x \leq a_4 \end{cases}$$

3.2. Ranking Procedure:(Patil et al. 2025)

Patil et al. (2025) have recently proposed a new approach to rank GTFNs. It is based on the intersection point of the trapezoid diagonals and is represented by a GTFN. This method has the potential advantage of being simpler than other conventionally used ranking methods; which prompted the authors of this article to apply the same in their current work. The said method shall hereafter be referred to as the *Point of Intersection of Diagonals* (PIOD) ranking method. The method is as follows:
Consider a GTFN $\tilde{A} = (a_1, a_2, a_3, a_4, w_A)$, forming a trapezoid ABCDA with coordinates $A = (a_1, 0)$, $B = (a_2, w)$, $C = (a_3, w)$, and $D = (a_4, 0)$ (Patil et al. 2025). The diagonals AC and BD of this trapezoid intersect at the points $\left(x_0 = \frac{a_3a_4 - a_1a_2}{(a_3+a_4)-(a_1+a_2)}, y_0 = \frac{w(a_4-a_1)}{(a_3+a_4)-(a_1+a_2)}\right)$.
For n trapezoidal numbers $\tilde{A}_i = (a_{1i}, a_{2i}, a_{3i}, a_{4i}; w_i)$; $i = 1, 2, ..., n$, the rank of $\tilde{A}_j$; $1 \leq j \leq n$ has been defined as

$$R\left(\tilde{A}_j\right) = (x_0)_{\tilde{A}_j}(y_0)_{\tilde{A}_j}\left(\frac{a_{1j}+a_{2j}+a_{3j}+a_{4j}}{4}\right)$$

The ranks obtained for a set of fuzzy numbers through the ranking method defined above; for two fuzzy numbers say $\tilde{A}$ and $\tilde{B}$, if $R\left(\tilde{A}\right) > R\left(\tilde{B}\right)$, then $\tilde{A} > \tilde{B}$. In this ranking method, if it is observed that two fuzzy numbers have equal ranks, it does not necessarily mean that the two fuzzy numbers are equal. We use the following to locate the greatest number of the two:

- For $\tilde{A} = (a_1, a_2, a_3, a_4, w_A)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, w_B)$, if $R\left(\tilde{A}\right) = R\left(\tilde{B}\right)$, then
  (1) $a_1 < b_1 \implies \tilde{A} < \tilde{B}$
  (2) if $a_1 = b_1$, then $a_2 < b_2 \implies \tilde{A} < \tilde{B}$
  (3) if $a_2 = b_2$, then $a_3 < b_3 \implies \tilde{A} < \tilde{B}$
  (4) if $a_3 = b_3$, then $a_4 < b_4 \implies \tilde{A} < \tilde{B}$
- For $\tilde{A}_i$ if $a_1 = a_2 = a_3 = a_4 = r$ (say), then $R\left(\tilde{A}_i\right) = r$

3.3. Arithmetic Operations:(Patil et al. 2025)

Arithmetic operations as considered on $\tilde{A}_1$ and $\tilde{A}_2$ defined as $(a_{11}, a_{12}, a_{13}, a_{14}, w_1)$ and $(a_{21}, a_{22}, a_{23}, a_{24}, w_2)$ respectively, are defined as follows:

- $\tilde{A}_1 + \tilde{A}_2 = (a_{11} + a_{21}, a_{12} + a_{22}, a_{13} + a_{23}, a_{14} + a_{24})$
- $\tilde{A}_1 - \tilde{A}_2 = (a_{11} - a_{24}, a_{12} - a_{23}, a_{13} - a_{22}, a_{14} - a_{21})$
- $\tilde{A}_1\tilde{A}_2 = (a, b, c, d)$ where
  $a = min\,(a_{11}a_{21}, a_{11}a_{24}, a_{14}a_{21}, a_{14}a_{24})$; $d = max\,(a_{11}a_{21}, a_{11}a_{24}, a_{14}a_{21}, a_{14}a_{24})$
  $b = min\,(a_{12}a_{22}, a_{12}a_{23}, a_{13}a_{22}, a_{13}a_{23})$; $c = max\,(a_{12}a_{22}, a_{12}a_{23}, a_{13}a_{22}, a_{13}a_{23})$

3.4. Fuzzy TP:

Consider an TP that involves m origins/sources $\{OR_i; i = 1, 2, ..., m\}$ and n destinations/sinks $\{DT_j; j = 1, 2, ..., n\}$. We consider the following:

- $\tilde{a}_i$: Availability of units at origin/source OR_i.
- $\tilde{b}_j$: Requirement of units at destination/sink DT_j.
- $\tilde{ct}_{ij}$: Transportation cost per unit from OR_i to DT_j.
- $\tilde{q}_{ij}$: Number of units, that is the quantity, to be transported from OR_i to DT_j.

Note:

- $\tilde{a}_i, \tilde{b}_j, \tilde{ct}_{ij}$, and $\tilde{q}_{ij}$ are fuzzy in nature.
- The transportation problem is said to be balanced if $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$. It is said to be unbalanced otherwise.

Table 1 gives the general representation of a fuzzy transportation problem with entities as defined above.

Table 1. Fuzzy Transportation Problem

	DT_1	DT_2	....	...	DT_n	Supply
OR_1	$\tilde{q}_{11}$ / $\tilde{ct}_{11}$	$\tilde{q}_{12}$ / $\tilde{ct}_{12}$	.... /	.... /	$\tilde{q}_{1n}$ / $\tilde{ct}_{1n}$	$\tilde{a}_1$
OR_2	$\tilde{q}_{21}$ / $\tilde{ct}_{21}$	$\tilde{q}_{22}$ / $\tilde{ct}_{22}$	.... /	.... /	$\tilde{q}_{2n}$ / $\tilde{ct}_{2n}$	$\tilde{a}_2$
OR_3	.... /	.... /	.... /	.... /	.... /	a_3
. .	.... /	.... /	.... /	.... /	.... /	. .
. OR_m	$\tilde{q}_{m1}$ / $\tilde{ct}_{m1}$	$\tilde{q}_{m2}$ / $\tilde{ct}_{m2}$	.... /	.... /	$\tilde{q}_{mn}$ / $\tilde{ct}_{mn}$	. $\tilde{a}_m$
Demand	$\tilde{b}_1$	$\tilde{b}_2$	....	....	$\tilde{b}_n$	

3.5. Fuzzy Transshipment Problem:

Consider a transshipment problem to be a modified or enhanced version of a TP, in both regular and fuzzy environments. Under this scenario, every source is also a sink, and vice-versa. Hence a TP with m sources and n sinks is now represented as one having $m+n$ sources and $m+n$ sinks. With sinks becoming possible sources and sources becoming possible sinks, the problem now is determining the minimum transportation cost under consideration of the newly introduced possible intermediate points. In order to incorporate sources as sinks and vice-versa, a buffer quantity of units, say $\tilde{B}$, needs to be taken into consideration to adjust the availabilities and requirements. Considering the case where the original TP is balanced, this buffer is usually taken as:

$$\tilde{B} = \sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$$

$\tilde{B}$ becomes the availability at each newly introduced source and the requirement of every newly introduced sink. The original sources and sink now have availabilities and requirements respectively as: $\tilde{a}_i + \tilde{B}$ and $\tilde{b}_j + \tilde{B}$. With these changes, it is to be noted that unit transportation costs from sink to sink and sink to source are either set by the personnel in-charge of formulating the problem at hand, or are set by the personnel responsible for decision-making. They may be made available separately. The costs along the diagonal of this table are set to zero since they essentially represent the cost of transporting a unit from a place to itself. With these suitable modifications, the transshipment problem can now be solved using any of the methods that can be used to solve a transportation problem. Table 2 gives the general representation of a transshipment problem in a fuzzy environment.

Table 2. Fuzzy Transshipment Problem

	OR_1	...	OR_m	DT_1	...	DT_n	Supply
OR_1	$\tilde{q}_{11}$ $\tilde{ct}_{11}$	... ...	$\tilde{q}_{1m}$ $\tilde{ct}_{1m}$	$\tilde{q}_{1,m+1}$ $\tilde{ct}_{1,m+1}$	... ...	$\tilde{q}_{1,m+n}$ $\tilde{ct}_{1,m+n}$	$\tilde{a}_1+\tilde{B}$
. .	... ...	... ...	... ...	... ...	... ...	... ...	. .
OR_m	$\tilde{q}_{m1}$ $\tilde{ct}_{m1}$	... ...	$\tilde{q}_{mm}$ $\tilde{ct}_{mm}$	$\tilde{q}_{m,m+1}$ $\tilde{ct}_{m,m+1}$	... ...	$\tilde{q}_{m,m+n}$ $\tilde{ct}_{m,m+n}$	$\tilde{a}_m + \tilde{B}$
DT_1	$\tilde{q}_{m+1,1}$ $\tilde{ct}_{m+1,1}$	... ...	$\tilde{q}_{m+1,m}$ $\tilde{ct}_{m+1,m}$	$\tilde{q}_{m+1,m+1}$ $\tilde{ct}_{m+1,m+1}$	... ...	$\tilde{q}_{m+1,m+n}$ $\tilde{ct}_{m+1,m+n}$	$\tilde{B}$
. .	... ...	... ...	... ...	... ...	... ...	... ...	. .
DT_n	$\tilde{q}_{m+n,1}$ $\tilde{ct}_{m+n,1}$	... ...	$\tilde{q}_{m+n,m}$ $\tilde{ct}_{m+n,m}$	$\tilde{q}_{m+n,m+1}$ $\tilde{q}_{m+n,m+1}$	... ...	$\tilde{q}_{m+n,m+n}$ $\tilde{ct}_{m+n,m+n}$	$\tilde{B}$
Demand	$\tilde{B}$	...	$\tilde{B}$	$\tilde{b}_{m+1}+\tilde{B}$	...	$\tilde{b}_{m+n}+\tilde{B}$	

3.6. Application of POID Ranking Method:

In this article, the authors have considered a balanced transshipment problem addressed by Gani et al. (2011), to demonstrate the efficacy of the POID ranking method. Ranks of all elements of the transshipment problem that are fuzzy in nature, are computed by the POID ranking method, and the rank-based Vogel's Approximation Method (VAM) is used to solve the considered balanced transshipment problem. The stepwise procedure applied is listed below.

Step 1: Compute the ranks of fuzzy costs corresponding to all cells of the transshipment table using the POID ranking method.

Step 2: Locate the minimum cost and the cost just lower than it, by ranks, from every row and every column of the transshipment table.

Step 3: Compute the row and column penalties as the absolute difference between the costs located in Step 2.

Step 4: Locate the row/column with the highest penalty. In this row/column, locate the cell having minimum cost.

Step 5: In the cell located in Step 4, make the highest possible allocation. This will be the lowest number, by means of ranks, out of the availability and requirement of the corresponding source and destination respectively.

Step 6: Cross off the exhausted supply (or demand) row (or column) as in the usual process of VAM.

Step 7: Repeat Steps 2 to 6 until all possible allocations are done.

Step 8: Calculate the cost contribution of every allocation $a \to b$ as $\tilde{q}_{ab}\tilde{ct}_{ab}$. It may be noted that diagonal entries are to be ignored.

Step 9: Calculate the fuzzy cost $\tilde{Z} = \sum\limits_{i \neq j} \tilde{q}_{ij}\tilde{ct}_{ij}$

Step 10: For $\tilde{Z} = (z_1, z_2, z_3, z_4, w_Z)$, say, calculate the crisp cost $Z = \left(\frac{z_1+2z_2+2z_3+z_4}{6}\right)$

Note: If tie(s) occur among the highest penalty in Step 3, they may be resolved as below:

(1) For the tied penalties:
  (a) Locate the minimum cost corresponding to each penalty value.
  (b) Select the cell having the least cost for allocation.
(2) If the minimum costs located above are all equal, select the cell where the highest possible allocation can be done.
(3) If the possible allocations are equal, then without loss of generality proceed with the cell having minimum cost in the left-most column (or top-most row) having tied penalties.

Table 3 presents the transshipment problem considered to demonstrate the efficacy of the suggested ranking method. Gani et al. (2011) have solved this problem using the Fuzzy Vam; by applying the ranking method suggested by S.-J. Chen and S.-M. Chen (2007). The crisp cost reached in the said article by this method is $Z = 7214.5$.

Table 3. Transshipment Costs, Supplies and Demands

	OR_1	OR_2	DT_1	DT_2	DT_3	Supply
OR_1	(0,0,0,0)	(73,74,75,76)	(9,10,11,12)	(11,12,13,14)	(23,24,25,26)	(592,596,600,604)
OR_2	(9,10,11,12)	(0,0,0,0)	(11,12,13,14)	(13,14,15,16)	(33,34,35,36)	(740,745,750,755)
DT_1	(11,12,13,14)	(23,24,25,26)	(0,0,0,0)	(31,32,33,34)	(9,10,11,12)	(444,447,450,453)
DT_2	(33,34,35,36)	(11,12,13,14)	(9,10,11,12)	(0,0,0,0)	(11,12,13,14)	(444,447,450,453)
DT_3	(53,54,55,56)	(63,64,65,66)	(73,74,75,76)	(11,12,13,14)	(0,0,0,0)	(444,447,450,453)
Demand	(444,447,450,453)	(444,447,450,453)	(592,596,600,604)	(592,596,600,604)	(592,596,600,604)	

This article uses the fuzzy VAM as well, and ranks are assigned to all fuzzy elements using the POID method of ranking. The resulting transshipment schedule and corresponding cost contributions are presented in Table 4. The crisp cost obtained by using the POID ranking method is $Z = 7201.67$.

Table 4. Allocations and Cost Contributions (POID Ranking method)

Route	Allocation	Unit Cost	Cost Contribution
$OR_3 \to DT_2$	(139,146,153,160)	(11,12,13,14)	(1529,1752,1989,2240)
$OR_2 \to DT_1$	(278,292,306,320)	(11,12,13,14)	(3058,3504,3978,4480)
$DT_1 \to DT_3$	(139,146,153,160)	(9,10,11,12)	(1251,1460,1683,1920)

Table 5. Comparative Summary of Results of Ranking Methods

Ranking Method Proposed by	Basis of the Ranking Method	Crisp Cost
S.-J. Chen and S.-M. Chen (2007)	Centroid formula	7214.5
Patil et al. (2025)	Point of intersection of diagonals of trapezoid	7201.67

4. Conclusion

The absence of straightforward and minimally intensive ranking methods acted as motivation for the authors to locate a differently defined ranking method. In their current work, the authors have identified the potential advantage that the POID ranking method (Patil et al. 2025) holds when applied to solve a transshipment problem under a fuzzy environment. This ranking method is simpler and less time consuming than the one proposed by S.-J. Chen and S.-M. Chen (2007), as it takes an approach that is computationally less intensive. The crisp costs obtained upon applying these two different ranking methods to solve the same fuzzy transshipment problem, are given in Table 5.

The authors, in their current work, have clearly demonstrated that the POID ranking method (Patil et al. 2025) is, in fact, better than the ranking method used by Gani et al. (2011) when applied to solving a fuzzy bi-index transshipment problem. This is established since an improvisation in the solution to a transshipment problem has been achieved through the application of the POID ranking method by obtaining a reduced transshipment cost.

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